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# Stress Wave propagation in Electro-Magneto-Elastic plate of arbitrary cross-sections

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**Abstract**

Wave propagation in electro-magneto-elastic plate of arbitrary cross-sections is studied using Fourier Expansion Collocation Method. A mathematical model is developed to study the wave propagation in a electro-magneto-elastic plate of arbitrary cross-sections using the three- dimensional theory of elasticity. The frequency equations are obtained from the boundary conditions, since the boundary is irregular in shape; it is difficult to satisfy the boundary conditions along the surface of the plate directly. Hence, the Fourier Expansion Collocation Method is applied along the boundary to satisfy the boundary conditions. The roots of the frequency equations are obtained by using the secant method, applicable for complex roots. The computed non-dimensional frequencies are plotted in the form of dispersion curves and its characteristics are analyzed.

**Keyword:** Vibrations of cylinders, Generalized thermo-elastic cylinder/plate, Mechanical vibrations, Stress-strain analysis, Electro-magneto-elastic materials, Piezoelectric plate.

## I. Introduction

The wave propagation in magneto-electro-elastic materials has gained considerable importance since last decade. The electro-magneto-elastic materials exhibit a desirable coupling effect between electric and magnetic fields, which are useful in smart structure applications. These materials have the capacity to convert one form of energy namely, magnetic, electric and mechanical energy to another form of energy. The composite consisting of piezoelectric and piezomagnetic components have found increasing application in engineering structures, particularly in smart/intelligent structure system. The magneto-electro-elastic materials are used as magnetic field probes, electric packing, acoustic, hydrophones, medical, ultrasonic image processing, sensors and actuators with the responsibility of magnetic-electro-mechanical energy conversion.

A method, for solving wave propagation in arbitrary and polygonal cross-sectional plates and to find out the phase velocities in different modes of vibrations namely longitudinal, torsional and flexural, by constructing frequency equations was devised by

Nagaya [1-3]. He formulated the Fourier expansion collocation method for this purpose and the same method is used in this problem.The three-dimensional behavior of magnetoelectroelastic laminates under simple support has been studied by Pan [4] and Pan and Heyliger [5]. An exact solution for magnetoelectroelastic laminates in cylindrical bending has also been obtained by Pan and Heyliger [6]. Pan and Han [7] studied the exact solution for functionally graded and layered magneto-electro-elastic plates. Feng and Pan [8] discussed the dynamic fracture behavior of an internal interfacial crack between two dissimilar magneto-electro-elastic plates. Buchanan [9] developed the free vibration of an infinite magneto-electro-elastic cylinder. Dai and Wang [10,11] have studied thermo-electro-elastic transient responses in piezoelectric hollow structures and hollow cylinder subjected to complex loadings. Later Kong et al [12] presented the thermo-magneto-dynamic stresses and perturbation of magnetic field vector in a non-homogeneous hollow cylinder. Annigeri et al [13-15], studied respectively, the free vibration of clamped-clamped magneto-electro-elastic cylindrical shells, free vibration behavior of multiphase and layered magneto-electro-elastic beam, free vibrations of simply supported layered and multiphase magneto-electro-elastic cylindrical shells. Hon et al [16] analyzed a point heat source on the surface of a semi-infinite transversely isotropic electro-magneto-thermo-elastic materials. Sharma and Mohinder Pal [17] developed the Rayleigh-Lamb waves in magneto-thermo-elastic homogeneous isotropic plate. Later Sharma and Thakur [18] studied the effect of rotation on Rayleigh-Lamb waves in magneto-thermo-elastic media. Gao and Noda [19] presented the thermal-induced interfacial cracking of magnetoelectroelastic materials. Bin et al [20] studied the wave propagation in non-homogeneous magneto-electro-elastic plates. Ponnusamy [21-23] have studied the wave propagation in generalized thermo-elastic cylinder of arbitrary cross section, thermoelastic and generalized thermo elastic plates of arbitrary and polygonal cross-sections respectively. Ponnusamy and Rajagopal [24,25] have studied, the wave propagation in a generalized thermo elastic solid cylinder of arbitrary cross-section and in a homogeneous transversely isotropic thermo elastic solid cylinder of polygonal cross-sections respectively using the Fourier expansion collocation method.

## 2. Formulation of the Problem

We consider a homogeneous transversely isotropic magneto-electro-elastic plate of arbitrary cross-sections. The system displacements and stresses are defined by the cylindrical co-ordinates r, $\theta$ and z. The governing equations of motion, electric and magnetic conduction equation in the absence of body force are

$$\sigma_{rr,r} + r^{-1}\sigma_{r\theta,\theta} + \sigma_{rz,z} + r^{-1}\left(\sigma_{rr} - \sigma_{\theta\theta}\right) = \rho u_{,tt}$$

$$\sigma_{r\theta,r} + r^{-1}\sigma_{\theta\theta,\theta} + \sigma_{\theta z,z} + 2r^{-1}\sigma_{r\theta} = \rho v_{,tt}$$

$$\sigma_{rz,r} + r^{-1}\sigma_{\theta z,\theta} + \sigma_{zz,z} + r^{-1}\sigma_{rz} = \rho w_{,tt} \quad . \tag{1}$$

The electric conduction equation is

$$D_{r,r} + r^{-1}D_{,r} + r^{-1}D_{\theta,\theta} + D_{z,z} = 0 \ . \tag{2}$$

The Magnetic conduction equation is

$$B_{r,r} + r^{-1}B_{r} + r^{-1}B_{\theta,\theta} + B_{z,z} = 0 \ . \tag{3}$$

Where

$$\sigma_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} + c_{13}e_{zz} - e_{31}E_z - q_{31}H_z$$

$$\sigma_{\theta\theta} = c_{12}e_{rr} + c_{11}e_{\theta\theta} + c_{13}e_{zz} - e_{31}E_z - q_{31}H_z$$

$$\sigma_{zz} = c_{13}e_{rr} + c_{13}e_{\theta\theta} + c_{33}e_{zz} - e_{33}E_z - q_{33}H_z$$

$$\sigma_{r\theta} = 2c_{66}e_{r\theta}$$

$$\sigma_{\theta z} = 2c_{44}e_{\theta z} - e_{15}E_{\theta} - q_{15}H_{\theta}$$

$$\sigma_{rz} = 2c_{44}e_{rz} - e_{15}E_r - q_{15}H_r \tag{4}$$

$$D_r = 2e_{15}e_{rz} + \varepsilon_{11}E_r + m_{11}H_r$$

$$D_{\theta} = 2e_{15}e_{\theta z} + \varepsilon_{11}E_{\theta} + m_{11}H_{\theta}$$

$$D_z = e_{31}\left(e_{rr} + e_{\theta\theta}\right) + e_{33}e_{zz} + \varepsilon_{33}E_z + m_{33}H_z \tag{5}$$

and

$$B_r = 2q_{15}e_{rz} + m_{11}E_r + \mu_{11}H_r$$

$$B_\theta = 2q_{15}e_{\theta z} + m_{11}E_\theta + \mu_{11}H_\theta$$

$$B_z = q_{31}\left(e_{rr} + e_{\theta\theta}\right) + q_{33}e_{zz} + m_{33}E_z + \mu_{33}H_z \ . \qquad (6)$$

Where $\sigma_{rr}, \sigma_{\theta\theta}, \sigma_{r\theta}, \sigma_{rz}, \sigma_{\theta z}$ are the stress components, $c_{11}, c_{12}, c_{13}, c_{33}, c_{44}$ and $c_{66} = \left(c_{11} - c_{12}\right)/2$ are elastic constants, $\varepsilon_{11}, \varepsilon_{33}$ are the dielectric constants, $\mu_{11}, \mu_{33}$ are the magnetic permeability coefficients, $e_{31}, e_{33}, e_{15}$ are the piezoelectric material coefficients, $q_{31}, q_{33}, q_{15}$ are the piezomagnetic material coefficients, $m_{11}, m_{33}$ are the magneto-electric material coefficients, $\rho$ is the mass density of the material, $D_r, D_\theta$ and $D_z$ are the electric displacements, $B_r, B_\theta$ and $B_z$ are the magnetic displacements components.

The strain $e_{ij}$ are related to the displacements corresponding to the cylindrical coordinates $(r, \theta, z)$ are given by

$$e_{rr} = u_{,r} \ , \ e_{\theta\theta} = r^{-1}\left(v_{,\theta} + u\right) \ , e_{zz} = w_{,z}$$

$$e_{\theta z} = \frac{1}{2}\left(v_{,z} + r^{-1}w_{,\theta}\right), \ e_{r\theta} = \frac{1}{2}\left(r^{-1}u_{,\theta} + v_{,r} - r^{-1}v\right), \ e_{rz} = \frac{1}{2}\left(u_{,z} + w_{,r}\right) \qquad (7)$$

where u, v and w are the mechanical displacements along the radial, circumferential and axial directions respectively.

The Electric field vector $E_i, \left(i = r, \theta, z\right)$ is related to the electric potential E as

$$E_r = -\frac{\partial E}{\partial r} \ , \ E_\theta = -\frac{1}{r}\frac{\partial E}{\partial \theta} \ \text{and} \ E_{,z} = -\frac{\partial E}{\partial z} \qquad (8)$$

Similarly, the magnetic field vector $H_i, \left(i = r, \theta, z\right)$ is related to the magnetic potential H as

$$H_r = -\frac{\partial H}{\partial r} \ , \ H_\theta = -\frac{1}{r}\frac{\partial H}{\partial \theta} \ \text{and} \ H_z = -\frac{\partial H}{\partial z} \qquad (9)$$

Substituting the Eqs. (4)-(9) in the Eqs. (1)-(3), we obtain the set of displacement equations as follows;

$$c_{11}\left(u_{,rr}+r^{-1}u_{,r}-r^{-2}u\right)+c_{66}r^{-2}u_{,\theta\theta}+c_{44}u_{,zz}+\left(c_{66}+c_{12}\right)r^{-1}v_{,r\theta}-\left(c_{11}+c_{66}\right)r^{-2}v_{,\theta}$$
$$+\left(c_{44}+c_{13}\right)w_{,rz}+\left(e_{31}+e_{15}\right)E_{,rz}+\left(q_{31}+q_{15}\right)H_{,rz}=\rho u_{,tt} \tag{10a}$$

$$\left(c_{66}+c_{12}\right)r^{-1}u_{,r\theta}+\left(c_{11}+c_{66}\right)r^{-2}u_{,\theta}+c_{66}\left(v_{,rr}+r^{-1}v_{,r}-r^{-2}v\right)+c_{44}v_{,zz}+c_{11}r^{-2}v_{,\theta\theta}$$
$$+\left(c_{44}+c_{13}\right)r^{-1}w_{,\theta z}+\left(e_{31}+e_{15}\right)r^{-1}E_{,\theta z}+\left(q_{31}+q_{15}\right)r^{-1}H_{,\theta z}=\rho v_{,tt} \tag{10b}$$

$$\left(c_{44}+c_{13}\right)\left(u_{,rz}+r^{-1}u_{,z}+r^{-1}v_{,\theta z}\right)+c_{44}\left(w_{,rr}+r^{-1}w_{,r}+w_{,\theta\theta}\right)+c_{33}w_{,zz}+e_{33}E_{,zz}$$
$$+q_{33}H_{,zz}+e_{15}\left(E_{,rr}+r^{-1}E_{,r}+r^{-2}E_{,\theta\theta}\right)+q_{15}\left(H_{,rr}+r^{-1}H_{,r}+r^{-2}H_{,\theta\theta}\right)=\rho w_{,tt} \tag{10c}$$

$$e_{15}\left(w_{,rr}+r^{-1}w_{,r}+r^{-2}w_{,\theta\theta}\right)+\left(e_{31}+e_{15}\right)\left(u_{,rz}+r^{-1}u_{,z}+r^{-1}v_{,\theta z}\right)+e_{33}w_{,zz}-\varepsilon_{33}E_{,zz}$$
$$-m_{33}H_{,zz}-\varepsilon_{11}\left(E_{,rr}+r^{-1}E_{,r}+r^{-2}E_{,\theta\theta}\right)-m_{11}\left(H_{,rr}+r^{-1}H_{,r}+r^{-2}H_{,\theta\theta}\right)=0 \tag{10d}$$

$$q_{15}\left(w_{,rr}+r^{-1}w_{,r}+r^{-2}w_{,\theta\theta}\right)+\left(q_{31}+q_{15}\right)\left(u_{,rz}+r^{-1}u_{,z}+r^{-1}v_{,\theta z}\right)+q_{33}w_{,zz}-m_{33}E_{,zz}$$
$$-\mu_{33}H_{,zz}-m_{11}\left(E_{,rr}+r^{-1}E_{,r}+r^{-2}E_{,\theta\theta}\right)-\mu_{11}\left(H_{,rr}+r^{-1}H_{,r}+r^{-2}H_{,\theta\theta}\right)=0 \tag{10e}$$

### 3. Solution of the Problem

The Eq. (10) is a coupled partial differential equation with three displacements and magnetic and electric conduction components. To uncouple the Eq. (10), we follow Sharma and Sharma [26] and seek the solutions in the following form

$$u=\sum\varepsilon_n\left[\left(r^{-1}\psi_{n,\theta}-\phi_{n,r}\right)+\left(r^{-1}\overline{\psi}_{n,\theta}-\overline{\phi}_{n,r}\right)\right]$$

$$v=\sum\varepsilon_n\left[\left(-r^{-1}\phi_{n,\theta}-\psi_{n,r}\right)+\left(-r^{-1}\overline{\phi}_{n,\theta}-\overline{\psi}_{n,r}\right)\right]$$

$$W=\sum\varepsilon_n\left[W_{n,z}+\overline{W}_{n,z}\right]$$

$$E=\sum\varepsilon_n\left[E_{n,z}+\overline{E}_{n,z}\right]$$

$$H=\sum\varepsilon_n\left[H_{n,z}+\overline{H}_{n,z}\right] \tag{11}$$

where $\varepsilon_n = 1/2$ for $n = 0$, $\varepsilon_n = 1$ for $n \geq 1, \phi_n(r,\theta)$, $\psi_n(r,\theta)$, $W_n(r,\theta)$, $E_n(r,\theta)$, $H_n(r,\theta)$ are the displacement potentials for the symmetric mode and $\bar{\phi}_n(r,\theta)$, $\bar{\psi}_n(r,\theta)$, $\bar{W}_n(r,\theta)$, $\bar{E}_n(r,\theta)$ and $\bar{H}_n(r,\theta)$ are the displacement potentials for the anti symmetric mode of vibrations.

Substituting the Eq.(11) in (10), we get

$$\left(c_{11}\nabla_1^2 + c_{44}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)\phi_n - (c_{13}+c_{44})\frac{\partial W_n}{\partial z} - (e_{31}+e_{15})\frac{\partial E_n}{\partial z} - (q_{31}+q_{15})\frac{\partial H_n}{\partial z} = 0 \tag{12a}$$

$$\left(c_{11}\nabla_1^2 + c_{44}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)\phi_n - (c_{13}+c_{44})\frac{\partial W_n}{\partial z} - (e_{31}+e_{15})\frac{\partial E_n}{\partial z} - (q_{31}+q_{15})\frac{\partial H_n}{\partial z} = 0 \tag{12b}$$

$$\begin{aligned}&\left(c_{44}\nabla_1^2 + c_{33}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)W_n - (c_{13}+c_{44})\frac{\partial}{\partial z}\nabla_1^2\phi_n + \left(e_{15}\nabla_1^2 + e_{33}\frac{\partial^2}{\partial z^2}\right)E_n \\ &+\left(q_{15}\nabla_1^2 + q_{33}\frac{\partial^2}{\partial z^2}\right)H_n = 0\end{aligned} \tag{12c}$$

$$\left(e_{15}\nabla_1^2 + e_{33}\frac{\partial^2}{\partial z^2}\right)W_n - (e_{31}+e_{15})\frac{\partial}{\partial z}\nabla_1^2\phi_n - \left(\varepsilon_{11}\nabla_1^2 + \varepsilon_{33}\frac{\partial^2}{\partial z^2}\right)E_n - \left(m_{11}\nabla_1^2 + m_{33}\frac{\partial^2}{\partial z^2}\right)H_n = 0 \tag{12d}$$

$$\left(q_{15}\nabla_1^2 + q_{33}\frac{\partial^2}{\partial z^2}\right)W_n - (q_{31}+q_{15})\nabla_1^2\phi_n - \left(m_{11}\nabla_1^2 + m_{33}\frac{\partial^2}{\partial z^2}\right)E_n - \left(\mu_{11}\nabla_1^2 + \mu_{33}\frac{\partial^2}{\partial z^2}\right)H_n = 0 \tag{12e}$$

and

$$\left(c_{66}\nabla_1^2 + c_{44}\frac{\partial^2}{\partial z^2} - \rho\frac{\partial^2}{\partial t^2}\right)\psi_n = 0 \tag{13}$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

The Eq. (13) gives purely transverse wave, which is not affected by the electric and magnetic field. This wave is polarized in the planes perpendicular to the z-axis and it may be referred as the simple harmonic wave. We assume that the disturbance is time harmonic through the factor $e^{i\omega t}$, $\omega$ is angular velocity and hence, the system of Eqs. (12a)-(12e) becomes

$$\left(c_{11}\nabla_1^2 + c_{44}\frac{\partial^2}{\partial z^2} + \rho\omega^2\right)\phi_n - \left(c_{13}+c_{44}\right)\frac{\partial W_n}{\partial z} - \left(e_{31}+e_{15}\right)\frac{\partial E_n}{\partial z} - \left(q_{31}+q_{15}\right)\frac{\partial H_n}{\partial z} = 0 \tag{14a}$$

$$\left(c_{11}\nabla_1^2 + c_{44}\frac{\partial^2}{\partial z^2} + \rho\omega^2\right)\phi_n - \left(c_{13}+c_{44}\right)\frac{\partial W_n}{\partial z} - \left(e_{31}+e_{15}\right)\frac{\partial E_n}{\partial z} - \left(q_{31}+q_{15}\right)\frac{\partial H_n}{\partial z} = 0 \tag{14b}$$

$$\begin{aligned}&\left(c_{44}\nabla_1^2 + c_{33}\frac{\partial^2}{\partial z^2} + \rho\omega^2\right)W_n - \left(c_{13}+c_{44}\right)\frac{\partial}{\partial z}\nabla_1^2\phi_n + \left(e_{15}\nabla_1^2 + e_{33}\frac{\partial^2}{\partial z^2}\right)E_n \\ &+\left(q_{15}\nabla_1^2 + q_{33}\frac{\partial^2}{\partial z^2}\right)H_n = 0\end{aligned} \tag{14c}$$

$$\left(e_{15}\nabla_1^2 + e_{33}\frac{\partial^2}{\partial z^2}\right)W_n - \left(e_{31}+e_{15}\right)\frac{\partial}{\partial z}\nabla_1^2\phi_n - \left(\varepsilon_{11}\nabla_1^2 + \varepsilon_{33}\frac{\partial^2}{\partial z^2}\right)E_n - \left(m_{11}\nabla_1^2 + m_{33}\frac{\partial^2}{\partial z^2}\right)H_n = 0 \tag{14d}$$

$$\left(q_{15}\nabla_1^2 + q_{33}\frac{\partial^2}{\partial z^2}\right)W_n - \left(q_{31}+q_{15}\right)\frac{\partial}{\partial z}\nabla_1^2\phi_n - \left(m_{11}\nabla_1^2 + m_{33}\frac{\partial^2}{\partial z^2}\right)E_n - \left(\mu_{11}\nabla_1^2 + \mu_{33}\frac{\partial^2}{\partial z^2}\right)H_n = 0 \tag{14e}$$

We consider the free vibration of arbitrary cross-sectional plate, so we assume that

$$\phi_n(r,\theta,z,t) = \phi_n(r)\cos(m\pi z/L)\cos n\theta$$

$$W_n(r,\theta,z,t) = W_n(r)\sin(m\pi z/L)\cos n\theta$$

$$E_n(r,\theta,z,t) = \left(\frac{c_{44}}{q_{33}}\right)E_n(r)\sin(m\pi z/L)\cos n\theta$$

$$H_n(r,\theta,z,t) = \left(\frac{c_{44}}{q_{33}}\right)H_n(r)\sin(m\pi z/L)\cos n\theta \tag{15}$$

and

$$\psi_n(r,\theta,z,t)=\psi_n(r)\cos(m\pi z/L)\sin n\theta \tag{16}$$

Introducing the dimensionless quantities such as

$$x=\frac{r}{a},\ t_L=\zeta a,\ \zeta=m\pi/L\ \ \bar{c}_{ij}=\frac{c_{ij}}{c_{44}},\ \bar{e}_{ij}=\frac{e_{ij}}{e_{33}},\ \bar{q}_{ij}=\frac{q_{ij}}{q_{33}},\ \Omega^2=\frac{\rho\omega^2a^2}{c_{44}},\ \bar{m}_{ij}=\frac{m_{ij}c_{44}}{q_{33}e_{33}},$$

$\bar{\mu}_{ij}=\dfrac{\mu_{ij}c_{44}}{q_{33}^2}$, $\bar{\varepsilon}_{ij}=\dfrac{\varepsilon_{ij}c_{44}}{e_{33}^2}$ , L is the length of the plate and using the Eqs. (15) and (16) in the Eqs. (14) and (13), we get

$$\left(\bar{c}_{11}\nabla_2^2-t_L^2+\Omega^2\right)\phi_n-\left(1+\bar{c}_{13}\right)t_LW_n-\left(\bar{e}_{31}+\bar{e}_{15}\right)t_LE_n-\left(\bar{q}_{31}+\bar{q}_{15}\right)t_LH_n=0$$

$$\left(\nabla_2^2+\Omega^2-\bar{c}_{33}t_L^2\right)W_n+\left(1+\bar{c}_{13}\right)t_L\nabla_2^2\phi_n+\left(\bar{e}_{15}\nabla_2^2-t_L^2\right)E_n+\left(\bar{q}_{15}\nabla_2^2-t_L^2\right)H_n=0$$

$$\left(\bar{e}_{15}\nabla_2^2-t_L^2\right)W_n+\left(\bar{e}_{31}+\bar{e}_{15}\right)t_L\nabla_2^2\phi_n+\left(\bar{\varepsilon}_{33}t_L^2-\bar{\varepsilon}_{11}\nabla_2^2\right)E_n+\left(\bar{m}_{33}t_L^2-\bar{m}_{11}\nabla_2^2\right)H_n=0$$

$$\left(\bar{q}_{15}\nabla_2^2-t_L^2\right)W_n+\left(\bar{q}_{31}+\bar{q}_{15}\right)t_L\nabla_2^2\phi_n+\left(\bar{m}_{33}t_L^2-\bar{m}_{11}\nabla_2^2\right)E_n+\left(\bar{\mu}_{33}t_L^2-\bar{\mu}_{11}\nabla_2^2\right)H_n=0 \tag{17}$$

and

$$\left(\bar{c}_{66}\nabla_2^2-t_L^2+\Omega^2\right)\psi_n=0 \tag{18}$$

Where $\nabla_2^2=\dfrac{\partial^2}{\partial x^2}+\dfrac{1}{x}\dfrac{\partial}{\partial x}-\dfrac{n^2}{x^2}$.

The Eq. (17) is a homogeneous linear equation which has a trivial solution to obtain the non-trivial solution, the determinant of the coefficient matrix is equal to zero. Thus we get

$$\begin{vmatrix} \left(\bar{c}_{11}\nabla_2^2 + g_1\right) & -g_2 t_L & -g_3 t_L & -g_4 t_L \\ g_2 t_L \nabla_2^2 & \left(\nabla_2^2 + g_5\right) & \left(\bar{e}_{15}\nabla_2^2 - t_L^2\right) & \left(\bar{q}_{15}\nabla_2^2 - t_L^2\right) \\ g_3 t_L \nabla_2^2 & \left(\bar{e}_{15}\nabla_2^2 - t_L^2\right) & \left(g_6 - \bar{\varepsilon}_{11}\nabla_2^2\right) & \left(g_7 - \bar{m}_{11}\nabla_2^2\right) \\ g_4 t_L \nabla_2^2 & \left(\bar{q}_{15}\nabla_2^2 - t_L^2\right) & \left(g_7 - \bar{m}_{11}\nabla_2^2\right) & \left(g_8 - \bar{\mu}_{11}\nabla_2^2\right) \end{vmatrix} \left(\phi_n, W_n, E_n, H_n\right) = 0 \tag{19}$$

where

$$g_1 = \Omega^2 - t_L^2,\ g_2 = 1 + \bar{c}_{13},\ g_3 = \left(\bar{e}_{31} + \bar{e}_{15}\right),\ g_4 = \left(\bar{q}_{31} + \bar{q}_{15}\right),\ g_5 = \left(\Omega^2 - \bar{c}_{33} t_L^2\right),$$

$$g_6 = \bar{\varepsilon}_{33} t_L^2,\ g_7 = \bar{m}_{33} t_L^2,\ \ g_8 = \bar{\mu}_{33} t_L^2$$

Evaluating the determinant given in Eq. (19), we obtain the partial differential equation of the form

$$\left(A\nabla_2^8 + B\nabla_2^6 + C\nabla_2^4 + D\nabla_2^2 + E\right)\left(\phi_n, W_n, E_n, H_n\right) = 0 \tag{20}$$

where

$$A = \bar{c}_{11}\left\{\bar{\varepsilon}_{11}\left[\bar{\mu}_{11} + \bar{q}_{15}^2\right] + \bar{\mu}_{11}\bar{e}_{15}^2 - \bar{m}_{11}\left[\bar{m}_{11} + 2\bar{e}_{15}\bar{q}_{15}\right]\right\}$$

$$
\begin{aligned}
B = \bar{c}_{11}\{ & -g_6\bar{\mu}_{11} - g_8\bar{\varepsilon}_{11} + 2g_7\bar{m}_{11} + g_5\left[\bar{\varepsilon}_{11}\bar{\mu}_{11} - \bar{m}_{11}^2\right] - \bar{e}_{15}\left[g_8\bar{e}_{15} + 2t_L^2\left(\bar{\mu}_{11} - \bar{m}_{11}\right)\right] \\
& + \bar{q}_{15}\left[-g_6\bar{q}_{15} + 2t_L^2\left(\bar{m}_{11} - \bar{\varepsilon}_{11}\right)\right] + 2g_7\bar{e}_{15}\bar{q}_{15}\} + g_1\left\{\bar{\varepsilon}_{11}\left[\bar{\mu}_{11} + \bar{q}_{15}^2\right] + \bar{\mu}_{11}\bar{e}_{15}^2 - \bar{m}_{11}\left[\bar{m}_{11} + 2\bar{e}_{15}\bar{q}_{15}\right]\right\} \\
& + g_2t_L^2\left\{g_2\left[\bar{\varepsilon}_{11}\bar{\mu}_{11} - \bar{m}_{11}^2\right] - \bar{e}_{15}\left[-g_3\bar{\mu}_{11} + g_4\bar{m}_{11}\right] + \bar{q}_{15}\left[-g_3\bar{m}_{11} + g_4\bar{\varepsilon}_{11}\right]\right\} \\
& - g_3t_L^2\left\{g_2\left[-\bar{\mu}_{11}\bar{e}_{15} + \bar{m}_{11}\bar{q}_{15}\right] + g_3\bar{\mu}_{11} + g_4\bar{m}_{11} + \bar{q}_{15}\left[g_3\bar{q}_{15} - g_4\bar{e}_{15}\right]\right\} \\
& + g_4t_L^2\left\{g_2\left[-\bar{m}_{11}\bar{e}_{15} + \bar{\varepsilon}_{11}\bar{q}_{15}\right] + g_3\bar{m}_{11} - g_4\bar{\varepsilon}_{11} + \bar{e}_{15}\left[g_3\bar{q}_{15} - g_4\bar{e}_{15}\right]\right\}
\end{aligned}
$$

$$
\begin{aligned}
C = \bar{c}_{11}\{ & g_6g_8 - g_7^2 + g_5\left[-g_6\bar{\mu}_{11} - g_8\bar{\varepsilon}_{11} + 2g_7\bar{m}_{11}\right] + 2t_L^2\left[-\bar{e}_{15}\left(-g_8 + g_7\right) + \bar{q}_{15}\left(-g_7 + g_8\right)\right] \\
& + t_L^4\left[\bar{\mu}_{11} - 2\bar{m}_{11} + \bar{\varepsilon}_{11}\right]\} + g_1\{-g_6\bar{\mu}_{11} - g_8\bar{\varepsilon}_{11} + 2g_7\bar{m}_{11} + g_5\left[\bar{\varepsilon}_{11}\bar{\mu}_{11} - \bar{m}_{11}^2\right] \\
& - \bar{e}_{15}\left[g_8\bar{e}_{15} + 2t_L^2\left(\bar{\mu}_{11} - \bar{m}_{11}\right)\right] + \bar{q}_{15}\left[-g_6\bar{q}_{15} + 2t_L^2\left(\bar{m}_{11} - \bar{\varepsilon}_{11}\right)\right] + 2g_7\bar{e}_{15}\bar{q}_{15}\} \\
& + g_2t_L^2\{g_2\left[-g_6\bar{\mu}_{11} - g_8\bar{\varepsilon}_{11} + 2g_7\bar{m}_{11}\right] - \bar{e}_{15}\left[g_3g_8 - g_4g_7\right] + \bar{q}_{15}\left[g_3g_7 - g_4g_6\right] \\
& + t_L^2\left[-g_3\bar{\mu}_{11} + g_4\bar{m}_{11} + g_3\bar{m}_{11} - g_4\bar{\varepsilon}_{11}\right]\} - g_3t_L^2\{g_2\left[g_8\bar{e}_{15} - g_7\bar{q}_{15} + t_L^2\left(\bar{\mu}_{11} - \bar{m}_{11}\right)\right] - g_3g_8 \\
& + g_4g_7 - g_5\left[-g_3\bar{\mu}_{11} + g_4\bar{m}_{11}\right] + \bar{q}_{15}t_L^2\left[-g_3 + g_4\right] - t_L^2\left[g_3\bar{q}_{15} - g_4\bar{e}_{15}\right]\} \\
& + g_4t_L^2\{g_2\left[g_7\bar{e}_{15} - g_6\bar{q}_{15} + t_L^2\left(\bar{m}_{11} - \bar{\varepsilon}_{11}\right)\right] - g_3g_7 + g_4g_6 - g_5\left[-g_3\bar{m}_{11} + g_4\bar{\varepsilon}_{11}\right] \\
& + \bar{e}_{15}t_L^2\left[-g_3 + g_4\right] - t_L^2\left[g_3\bar{q}_{15} - g_4\bar{e}_{15}\right]\}
\end{aligned}
$$

$$
\begin{aligned}
D = \bar{c}_{11} & \left\{g_5\left[g_6g_8 - g_7^2\right] + t_L^4\left[-g_8 + 2g_7 - g_6\right]\right\} + g_4t_L^2\{g_2t_L^2\left[-g_7 + g_6\right] \\
& - g_5\left[g_3g_7 - g_4g_6\right] - t_L^4\left[-g_3 + g_4\right]\} + g_1\{g_6g_8 - g_7^2 + g_5\left[-g_6\bar{\mu}_{11} - g_8\bar{\varepsilon}_{11} + 2g_7\bar{m}_{11}\right] \\
& + 2t_L^2\left[-\bar{e}_{15}\left(-g_8 + g_7\right) + \bar{q}_{15}\left(-g_7 + g_8\right)\right] + t_L^4\left[\bar{\mu}_{11} - 2\bar{m}_{11} + \bar{\varepsilon}_{11}\right]\} \\
& + g_2t_L^2\left\{g_2\left[g_6g_8 - g_7^2\right] + t_L^2\left[g_3g_8 - g_4g_7 - g_3g_7 + g_4g_6\right]\right\} \\
& - g_3t_L^2\left\{g_2t_L^2\left[-g_8 + g_7\right] - g_5\left[g_3g_8 - g_4g_7\right] - t_L^4\left[-g_3 + g_4\right]\right\}
\end{aligned}
$$

$$
E = g_1\left\{g_5\left[g_6g_8 - g_7^2\right] + t_L^4\left[-g_8 + 2g_7 - g_6\right]\right\}
$$

Solving the Eq. (20), the solution for the symmetric mode obtained as

$$
\phi_n^* = \sum_{i=1}^{4} A_{in} J_n\left(\alpha_i r\right)\cos n\theta
$$

$$
W_n^* = \sum_{i=1}^{4} a_i A_{in} J_n\left(\alpha_i r\right)\cos n\theta
$$

$$
E_n^* = \sum_{i=1}^{4} b_i A_{in} J_n\left(\alpha_i r\right)\cos n\theta
$$

$$H_n^* = \sum_{i=1}^{4} c_i A_{in} J_n(\alpha_i r)\cos n\theta \tag{21a}$$

The solutions to the anti symmetric modes of vibrations $\overline{\phi}_n^*$, $\overline{W}_n^*$, $\overline{E}_n^*$, $\overline{H}_n^*$ are obtained by changing $\cos n\theta$ by $\sin n\theta$ in the Eq. (21a), we get

$$\overline{\phi}_n^* = \sum_{i=1}^{4} \overline{A}_{in} J_n(\alpha_i r)\sin n\theta$$

$$\overline{W}_n^* = \sum_{i=1}^{4} a_i \overline{A}_{in} J_n(\alpha_i r)\sin n\theta$$

$$\overline{E}_n^* = \sum_{i=1}^{4} b_i \overline{A}_{in} J_n(\alpha_i r)\sin n\theta$$

$$\overline{H}_n^* = \sum_{i=1}^{4} c_i \overline{A}_{in} J_n(\alpha_i r)\sin n\theta \quad . \tag{21b}$$

where $J_n$ is the Bessel function of first kind of order n .The constants $a_i$, $b_i$ and $c_i$ defined in the Eq. (21) is calculated using the following equations

$$-g_2 t_L a_i - g_3 t_L b_i - g_4 t_L c_i = \overline{c}_{11}\alpha_i^2 - g_1$$

$$\left(-\alpha_i^2 + g_5\right)a_i - \left(\overline{e}_{15}\alpha_i^2 + t_L^2\right)b_i - \left(\overline{q}_{15}\alpha_i^2 + t_L^2\right)c_i = g_2 t_L \alpha_i^2$$

$$-\left(\overline{e}_{15}\alpha_i^2 + t_L^2\right)a_i + \left(g_6 + \overline{\varepsilon}_{11}\alpha_i^2\right)b_i + \left(g_7 + \overline{m}_{11}\alpha_i^2\right)c_i = g_3 t_L \alpha_i^2$$

$$-\left(\overline{q}_{15}\alpha_i^2 + t_L^2\right)a_i + \left(g_7 + \overline{m}_{11}\alpha_i^2\right)b_i + \left(g_8 + \overline{\mu}_{11}\alpha_i^2\right)c_i = g_4 t_L \alpha_i^2 \tag{22}$$

Solving the Eq. (22), we obtain

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$$a_i = \frac{-g_3\left(g_7 + \bar{m}_{11}\alpha_i^2\right) + g_4\left(g_6 + \bar{\varepsilon}_{11}\alpha_i^2\right)}{-g_2\left(g_6 + \bar{\varepsilon}_{11}\alpha_i^2\right) - g_3\left(\bar{e}_{15}\alpha_i^2 + t_L^2\right)}$$

$$b_i = \frac{g_3 g_4 t_L^2 \alpha_i^2 + \left(\bar{c}_{11}\alpha_i^2 - g_1\right)\left(g_7 + \bar{m}_{11}\alpha_i^2\right)}{t_L\left[g_2\left(g_6 + \bar{\varepsilon}_{11}\alpha_i^2\right) + g_3\left(\bar{e}_{15}\alpha_i^2 + t_L^2\right)\right]}$$

$$c_i = \frac{\left(\bar{c}_{11}\alpha_i^2 - g_1\right)\left(\bar{e}_{15}\alpha_i^2 + t_L^2\right) - g_2 g_3 t_L^2 \alpha_i^2}{t_L\left[g_2\left(g_6 + \bar{\varepsilon}_{11}\alpha_i^2\right) + g_3\left(\bar{e}_{15}\alpha_i^2 + t_L^2\right)\right]}$$

Solving the Eq. (13), we obtain the solution for symmetric mode as

$$\psi_n^* = A_5 J_n\left(\alpha_5 r\right)\sin n\theta \tag{23a}$$

and the solution for anti symmetric mode is $\bar{\psi}_n^*$ obtained by changing $\sin n\theta$ by $\cos n\theta$ in the Eq. (23a), we get

$$\bar{\psi}_n^* = \bar{A}_5 J_n\left(\alpha_5 r\right)\cos n\theta \tag{23b}$$

Where $J_n$ is the Bessel functions of first kind of order $n$, and $\alpha_5^2 = \left(t_L^2 - \Omega^2\right)/c_{66}$. If $\left(\alpha_i a\right)^2 < 0\left(i = 1,2,3,4,5\right)$, then the Bessel function $J_n$ is replaced by the modified Bessel function $I_n$.

## 4. Boundary conditions and frequency Equations

In this problem, the vibration of arbitrary cross-sectional plate is considered. Since the boundary is irregular in shape, it is difficult to satisfy the boundary conditions along the surface of the plate directly. Hence, the Fourier expansion collocation method is applied to satisfy the boundary conditions. For the plate, the normal stress $\sigma_{xx}$ and shearing stresses $\sigma_{xy}, \sigma_{xz}$, the electric field $D_x$ and the magnetic field $B_x$ is equal to zero for stress free boundary. Thus the following types of boundary conditions are assumed for the plate of arbitrary cross-section is

$$\left(\sigma_{xx}\right)_i = \left(\sigma_{xy}\right)_i = \left(\sigma_{xz}\right)_i = \left(D_x\right)_i = \left(B_x\right)_i = 0 \tag{24}$$

where $\left(\ \right)_i$ is the value at the $i-th$ segment of the boundary, if the angle $\gamma_i$ between the normal to the segment and the reference axis is assumed to be constant, then the transformed expression for the stresses are given by

$$\sigma_{xx} = \left(c_{11}\cos^2\left(\theta-\gamma_i\right) + c_{12}\sin^2\left(\theta-\gamma_i\right)\right)u_{,r} + r^{-1}\left(c_{11}\sin^2\left(\theta-\gamma_i\right) + c_{12}\cos^2\left(\theta-\gamma_i\right)\right)\left(u+v_{,\theta}\right)$$
$$+c_{66}\left(r^{-1}\left(v-u_{,\theta}\right)-v_{,r}\right)\sin 2\left(\theta-\gamma_i\right) + c_{13}W_{,z} + e_{31}E_{,z} + q_{31}H_{,z} = 0$$

$$\sigma_{xy} = c_{66}\left(\left(u_{,r} - r^{-1}\left(v_{,\theta}+u\right)\right)\sin 2\left(\theta-\gamma_i\right) + \left(r^{-1}\left(u_{,\theta}-v\right)+v_{,r}\right)\cos 2\left(\theta-\gamma_i\right)\right) = 0$$

$$\sigma_{xz} = c_{44}\left(\left(u_{,z}+W_{,r}\right)\cos\left(\theta-\gamma_i\right) - \left(v_{,z}+r^{-1}W_{,\theta}\right)\sin\left(\theta-\gamma_i\right)\right) + e_{15}E_{,r} + q_{15}H_{,r} = 0$$
$$D_x = e_{15}\left(u_{,z}+W_{,r}\right) - \varepsilon_{11}E_{,r} - m_{11}H_{,r} = 0$$

$$B_x = q_{15}\left(u_{,z}+W_{,r}\right) - m_{11}E_{,r} - \mu_{11}H_{,r} = 0 \tag{25}$$

Substituting the Eqs. (21) and (23) in the Eq. (24) and performing the Fourier series expansion to Eq.(24) along the boundary as discussed in Ponnusamy [21-23], the boundary condition along the boundary of the surfaces are expanded in the form of double Fourier series. When the plate is symmetric about more than one axis, the boundary conditions in the case of symmetric mode can be written in the form of a matrix as given below:

$$\begin{bmatrix}
E_{00}^1 & E_{00}^2 & E_{00}^3 & E_{00}^4 & 0 & E_{01}^1 & \cdots & E_{0N}^1 & E_{01}^2 & \cdots & E_{0N}^2 & E_{01}^3 & \cdots & E_{0N}^3 & E_{01}^4 & \cdots & E_{0N}^4 & E_{01}^5 & \cdots & E_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
E_{N0}^1 & E_{N0}^2 & E_{N0}^3 & E_{N0}^4 & 0 & E_{N1}^1 & \cdots & E_{NN}^1 & E_{N1}^2 & \cdots & E_{NN}^2 & E_{N1}^3 & \cdots & E_{NN}^3 & E_{N1}^4 & \cdots & E_{NN}^4 & E_{N1}^5 & \cdots & E_{NN}^5 \\
F_{00}^1 & F_{00}^2 & F_{00}^3 & F_{00}^4 & 0 & F_{01}^1 & \cdots & F_{0N}^1 & F_{01}^2 & \cdots & F_{0N}^2 & F_{01}^3 & \cdots & F_{0N}^3 & F_{01}^4 & \cdots & F_{0N}^4 & F_{01}^5 & \cdots & F_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
F_{N0}^1 & F_{N0}^2 & F_{N0}^3 & F_{N0}^4 & 0 & F_{N1}^1 & \cdots & F_{NN}^1 & F_{N1}^2 & \cdots & F_{NN}^2 & F_{N1}^3 & \cdots & F_{NN}^3 & F_{N1}^4 & \cdots & F_{NN}^4 & F_{N1}^5 & \cdots & F_{NN}^5 \\
G_{00}^1 & G_{00}^2 & G_{00}^3 & G_{00}^4 & 0 & G_{01}^1 & \cdots & G_{0N}^1 & G_{01}^2 & \cdots & G_{0N}^2 & G_{01}^3 & \cdots & G_{0N}^3 & G_{01}^4 & \cdots & G_{0N}^4 & G_{01}^5 & \cdots & G_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
G_{N0}^1 & G_{N0}^2 & G_{N0}^3 & G_{N0}^4 & 0 & G_{N1}^1 & \cdots & G_{NN}^1 & G_{N1}^2 & \cdots & G_{NN}^2 & G_{N1}^3 & \cdots & G_{NN}^3 & G_{N1}^4 & \cdots & G_{NN}^4 & G_{N1}^5 & \cdots & G_{NN}^5 \\
H_{00}^1 & H_{00}^2 & H_{00}^3 & H_{00}^4 & 0 & H_{01}^1 & \cdots & H_{0N}^1 & H_{01}^2 & \cdots & H_{0N}^2 & H_{01}^3 & \cdots & H_{0N}^3 & H_{01}^4 & \cdots & H_{0N}^4 & H_{01}^5 & \cdots & H_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
H_{N0}^1 & H_{N0}^2 & H_{N0}^3 & H_{N0}^4 & 0 & H_{N1}^1 & \cdots & H_{NN}^1 & H_{N1}^2 & \cdots & H_{NN}^2 & H_{N1}^3 & \cdots & H_{NN}^3 & H_{N1}^4 & \cdots & H_{NN}^4 & H_{N1}^5 & \cdots & H_{NN}^5 \\
I_{00}^1 & I_{00}^2 & I_{00}^3 & I_{00}^4 & 0 & I_{01}^1 & \cdots & I_{0N}^1 & I_{01}^2 & \cdots & I_{0N}^2 & I_{01}^3 & \cdots & I_{0N}^3 & I_{01}^4 & \cdots & I_{0N}^4 & I_{01}^5 & \cdots & I_{0N}^5 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
I_{N0}^1 & I_{N0}^2 & I_{N0}^3 & I_{N0}^4 & 0 & I_{N1}^1 & \cdots & I_{NN}^1 & I_{N1}^2 & \cdots & I_{NN}^2 & I_{N1}^3 & \cdots & I_{NN}^3 & I_{N1}^4 & \cdots & I_{NN}^4 & I_{N1}^5 & \cdots & I_{NN}^5
\end{bmatrix}
\begin{bmatrix} A_{10} \\ A_{20} \\ A_{30} \\ A_{40} \\ A_{50} \\ A_{11} \\ \vdots \\ A_{1N} \\ A_{21} \\ \vdots \\ A_{2N} \\ \vdots \\ A_{51} \\ \vdots \\ A_{5N} \end{bmatrix} = 0 \tag{26}$$

where

$$E_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} e_n^j\left(R_i,\theta\right)\cos m\theta d\theta\,,\; F_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} f_n^j\left(R_i,\theta\right)\sin m\theta d\theta$$

$$G_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} g_n^j\left(R_i,\theta\right)\cos m\theta d\theta,\; H_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} h_n^j\left(R_i,\theta\right)\cos m\theta d\theta,$$

$$I_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} i_n^j\left(R_i,\theta\right)\cos m\theta d\theta \tag{27}$$

The coefficients $e_n^i$ □ $i_n^i$ are given in the Appendix A.

Similarly, the matrix for the anti symmetric mode is obtained as

$$\begin{bmatrix}
\bar{E}_{10}^{5} & \bar{E}_{11}^{1} & \cdots & \bar{E}_{1N}^{1} & \bar{E}_{11}^{2} & \cdots & \bar{E}_{1N}^{2} & \bar{E}_{11}^{3} & \cdots & \bar{E}_{1N}^{3} & \bar{E}_{11}^{4} & \cdots & \bar{E}_{1N}^{4} & \bar{E}_{11}^{5} & \cdots & \bar{E}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\bar{E}_{N0}^{5} & \bar{E}_{N1}^{1} & \cdots & \bar{E}_{NN}^{1} & \bar{E}_{N1}^{2} & \cdots & \bar{E}_{NN}^{2} & \bar{E}_{N1}^{3} & \cdots & \bar{E}_{NN}^{3} & \bar{E}_{N1}^{4} & \cdots & \bar{E}_{NN}^{4} & \bar{E}_{N1}^{5} & \cdots & \bar{E}_{NN}^{5} \\
\bar{F}_{10}^{5} & \bar{F}_{11}^{1} & \cdots & \bar{F}_{1N}^{1} & \bar{F}_{11}^{2} & \cdots & \bar{F}_{1N}^{2} & \bar{F}_{11}^{3} & \cdots & \bar{F}_{1N}^{3} & \bar{F}_{11}^{4} & \cdots & \bar{F}_{1N}^{4} & \bar{F}_{11}^{5} & \cdots & \bar{F}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\bar{F}_{N0}^{5} & \bar{F}_{N1}^{1} & \cdots & \bar{F}_{NN}^{1} & \bar{F}_{N1}^{2} & \cdots & \bar{F}_{NN}^{2} & \bar{F}_{N1}^{3} & \cdots & \bar{F}_{NN}^{3} & \bar{F}_{N1}^{4} & \cdots & \bar{F}_{NN}^{4} & \bar{F}_{N1}^{5} & \cdots & \bar{F}_{NN}^{5} \\
\bar{G}_{10}^{5} & \bar{G}_{11}^{1} & \cdots & \bar{G}_{1N}^{1} & \bar{G}_{11}^{2} & \cdots & \bar{G}_{1N}^{2} & \bar{G}_{11}^{3} & \cdots & \bar{G}_{1N}^{3} & \bar{G}_{11}^{4} & \cdots & \bar{G}_{1N}^{4} & \bar{G}_{11}^{5} & \cdots & \bar{G}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\bar{G}_{N0}^{5} & \bar{G}_{N1}^{1} & \cdots & \bar{G}_{NN}^{1} & \bar{G}_{N1}^{2} & \cdots & \bar{G}_{NN}^{2} & \bar{G}_{N1}^{3} & \cdots & \bar{G}_{NN}^{3} & \bar{G}_{N1}^{4} & \cdots & \bar{G}_{NN}^{4} & \bar{G}_{N1}^{5} & \cdots & \bar{G}_{NN}^{5} \\
\bar{H}_{10}^{5} & \bar{H}_{11}^{1} & \cdots & \bar{H}_{1N}^{1} & \bar{H}_{11}^{2} & \cdots & \bar{H}_{1N}^{2} & \bar{H}_{11}^{3} & \cdots & \bar{H}_{1N}^{3} & \bar{H}_{11}^{4} & \cdots & \bar{H}_{1N}^{4} & \bar{H}_{11}^{5} & \cdots & \bar{H}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\bar{H}_{N0}^{5} & \bar{H}_{N1}^{1} & \cdots & \bar{H}_{NN}^{1} & \bar{H}_{N1}^{2} & \cdots & \bar{H}_{NN}^{2} & \bar{H}_{N1}^{3} & \cdots & \bar{H}_{NN}^{3} & \bar{H}_{N1}^{4} & \cdots & \bar{H}_{NN}^{4} & \bar{H}_{N1}^{5} & \cdots & \bar{H}_{NN}^{5} \\
\bar{I}_{10}^{5} & \bar{I}_{11}^{1} & \cdots & \bar{I}_{1N}^{1} & \bar{I}_{11}^{2} & \cdots & \bar{I}_{1N}^{2} & \bar{I}_{11}^{3} & \cdots & \bar{I}_{1N}^{3} & \bar{I}_{11}^{4} & \cdots & \bar{I}_{1N}^{4} & \bar{I}_{11}^{5} & \cdots & \bar{I}_{1N}^{5} \\
\vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots & \vdots & & \vdots \\
\bar{I}_{N0}^{5} & \bar{I}_{N1}^{1} & \cdots & \bar{I}_{NN}^{1} & \bar{I}_{N1}^{2} & \cdots & \bar{I}_{NN}^{2} & \bar{I}_{N1}^{3} & \cdots & \bar{I}_{NN}^{3} & \bar{I}_{N1}^{4} & \cdots & \bar{I}_{NN}^{4} & \bar{I}_{N1}^{5} & \cdots & \bar{I}_{NN}^{5}
\end{bmatrix}
\begin{bmatrix}
\bar{A}_{50} \\ \bar{A}_{11} \\ \vdots \\ \bar{A}_{1N} \\ \bar{A}_{21} \\ \vdots \\ \bar{A}_{2N} \\ \bar{A}_{31} \\ \vdots \\ \bar{A}_{3N} \\ \bar{A}_{41} \\ \vdots \\ \bar{A}_{4N} \\ \bar{A}_{51} \\ \vdots \\ \bar{A}_{5N}
\end{bmatrix} = 0 \tag{28}$$

where

$$\bar{E}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \bar{e}_n^j\left(R_i,\theta\right)\sin m\theta d\theta,\; \bar{F}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \bar{f}_n^j\left(R_i,\theta\right)\cos m\theta d\theta$$

$$\bar{G}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \bar{g}_n^j\left(R_i,\theta\right)\sin m\theta d\theta,\; \bar{H}_{mn}^{j}=\left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \bar{h}_n^j\left(R_i,\theta\right)\sin m\theta d\theta$$

$$\bar{I}^{j}_{mn} = \left(\frac{2\varepsilon_n}{\pi}\right)\sum_{i=1}^{I}\int_{\theta_{i-1}}^{\theta_i} \bar{i}^{j}_{n}\left(R_i,\theta\right)\sin m\theta d\theta \tag{29}$$

where j=1,2,3,4 and 5, I is the number of segments, $R_i$ is the coordinate r at the boundary and N is the number of truncation of the Fourier series. The frequency equations are obtained by truncating the series to N+1 terms, and equating the determinant of the coefficients of the amplitude $A_{in} = 0$ and $\bar{A}_{in} = 0$ (i=1,2,3,4 and 5), for symmetric and anti symmetric modes of vibrations.

**5. Solid circular Plate**

The frequency equation for solid circular plate can be written in the form

$$\left|A\right| = 0 \tag{30}$$

where $A$ is the $5\times5$ matrix with elements $a_{ij}\left(i, j = 1,2,3,4,5\right)$ are given by

$$a_{1i} = [-2\bar{c}_{66}\left\{n(n-1)J_n\left(\alpha_i ax\right)+\left(\alpha_i ax\right)J_{n+1}\left(\alpha_i ax\right)\right\}+x^2[\bar{c}_{11}\left(\alpha_i a\right)+t_L\left(\bar{c}_{13}a_i+\bar{e}_{31}b_i+\bar{q}_{31}c_i\right)J_n\left(\alpha_i ax\right)], i = 1,2,3,4$$

$$a_{15} = 2n\bar{c}_{66}\left\{n(n-1)J_n\left(\alpha_5 ax\right)-\left(\alpha_5 ax\right)J_{n+1}\left(\alpha_5 ax\right)\right\}$$

$$a_{2i} = \left\{2n\left(n-1\right)+J_n\left(\alpha_i ax\right)+\left(\alpha_i ax\right)J_{n+1}\left(\alpha_i ax\right)\right\}, i = 1,2,3,4$$

$$a_{25} = \bar{c}_{66}\left[\left\{-2n\left(n-1\right)+J_n\left(\alpha_5 ax\right)+\left(\alpha_5 ax\right)J_{n+1}\left(\alpha_5 ax\right)\right\}+\left(\alpha_5 ax\right)^2 J_n\left(\alpha_i ax\right)\right]$$

$$a_{3i} = \left[\left(t_L+a_i\right)+\bar{e}_{15}b_i+\bar{q}_{15}c_i\right]\left\{nJ_n\left(\alpha_i ax\right)-\left(\alpha_i ax\right)J_{n+1}\left(\alpha_i ax\right)\right\}, i = 1,2,3,4$$

$$a_{35} = -nt_L J_n\left(\alpha_5 ax\right)$$

$$a_{4i} = \left[\bar{e}_{15}\left(t_L+a_i\right)-\bar{\varepsilon}_{11}b_i-\bar{m}_{11}c_i\right]\left\{nJ_n\left(\alpha_i ax\right)-\left(\alpha_i ax\right)J_{n+1}\left(\alpha_i ax\right)\right\}, i = 1,2,3,4$$

$$a_{45} = -\bar{e}_{15}nt_L J_n\left(a_5 ax\right)$$

$$a_{5i} = \left[\bar{q}_{15}\left(t_L+a_i\right)-\bar{m}_{11}b_i-\bar{\mu}_{11}c_i\right]\left\{nJ_n\left(\alpha_i ax\right)-\left(\alpha_i ax\right)J_{n+1}\left(\alpha_i ax\right)\right\}, i = 1,2,3,4.$$

$$a_{55} = -\bar{q}_{15} n t_L J_n(a_5 a x)$$

## 6. Numerical results and Discussions

The electro-magnetic material constants based on graphical results of Aboudi, 2001[27] used for the numerical calculations is given in the Table 1.

**Table. 1** The material properties of the electro-magnetic material based on graphical results of Aboudi [27] composites

| $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{33}$ | $c_{44}$ | $c_{66}$ |
|---|---|---|---|---|---|
| 218 | 120 | 120 | 215 | 50 | 49 |
| $e_{15}$ | $e_{31}$ | $e_{33}$ | $q_{15}$ | $q_{31}$ | $q_{33}$ |
| 0 | -2.5 | 7.5 | 200 | 265 | 345 |
| $\varepsilon_{11}$ | $\varepsilon_{33}$ | $\mu_{11}$ | $\mu_{33}$ | $m_{11}$ | $m_{33}$ |
| 0.4 | 5.8 | -200 | 95 | 0.0074 | 2.82 |

Units: $c_{ij}\left(10^9 N/m^2\right), \varepsilon_{ij}\left(10^{-9} C/Vm\right), e_{ij}\left(C/m^2\right)$
$q_{ij}\left(N/Am\right), \mu_{ij}\left(10^{-6} Ns^2/C^2\right), m_{ij}\left(10^{-9} Ns/VC\right)$

In the numerical calculation, the angle $\theta$ is taken as an independent variable and the coordinate $R_i$ at the $i-th$ segment of the boundary is expressed in terms of $\theta$. Substituting $R_i$ and the angle $\gamma_i$, between the reference axis and the normal to the $i-th$ boundary line, the integrations of the Fourier coefficients $e_n^i$, $f_n^i$, $g_n^i$, $h_n^i$, $i_n^i$, $\bar{e}_n^i$, $\bar{f}_n^i$, $\bar{g}_n^i$, $\bar{h}_n^i$ and $\bar{i}_n^i$ can be expressed in terms of the angle $\theta$. Using these coefficients in to the Eqs. (27) and (29), the frequencies are obtained for electro-magneto-elastic plate of arbitrary cross-sections. In the present problem, there are two kinds of basic independent modes of wave propagation have been considered, namely, the longitudinal and flexural anti symmetric modes of vibrations.

### 6.1 Elliptic cross-section

The elliptic cross section of a plate is shown in Figure 1 and its geometric relations used for numerical calculations given below are due to Nagaya [2] as,

$$R_i/b = (a/b)/\left(\cos^2\theta + (a/b)^2 \sin^2\theta\right)^{1/2}, \quad \gamma_i = \pi/2 - \tan^{-1}\left((b/a)^2/\tan\theta_i^*\right), \text{for } \theta_i^* < \pi/2$$

$$\gamma_i = \pi/2 \text{, for } \theta_i^* = \pi/2 \text{, } \gamma_i = \pi\Big/2 + \tan^{-1}\left(\left(b/a\right)^2\Big/\left|\tan\theta_i^*\right|\right) \text{, for } \theta_i^* > \pi/2. \qquad (31)$$

where a is the semimajor axis and b is the semiminor axis of the elliptic plate and $\theta_i^* = \left(\theta_i + \theta_{i-1}\right)/2$, $R_i$ is the coordinate r at the boundary, $\gamma_i$ is the angle between the normal to the segment and the reference axis at the $i-th$ boundary.

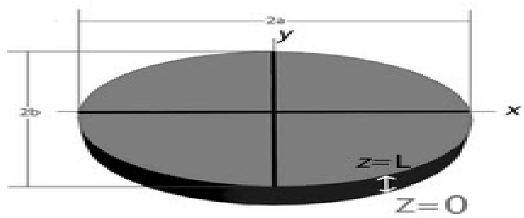


**Figure 1 Elliptic cross-sectional plate**

### 6.2 Parabolic cross-section

The geometry of the parabolic cross section is given in Figure 2. The geometric relations of the parabolic cross section given by Nagaya [1] are as follows:

$$R_i/c = \left(e/c\right)^2 \left\{-\cos\theta + \left[\cos^2\theta + 2\left(c/e\right)^2 \sin^2\theta\right]^{1/2}\right\}\Big/2\sin^2\theta$$

$$\gamma_i = \frac{\pi}{2} - \tan^{-1}\left[\left(e/c\right)^2 \frac{c}{R_i\left(\theta_i^*\right)} \frac{1}{2\sin\theta_i^*}\right]$$

$\delta = \pi - \tan^{-1}\left(2e/c\right)$ for the parabolic curve, and (32a)

$R_i/c = -1/2\cos\theta$, $\gamma_l = \pi$ for the straight line boundary (32b)

where $\theta_i^* = \left(\theta_i + \theta_{i-1}\right)/2$, $R_i$ is the coordinate $r$ at the boundary and $\gamma_i$ is the angle between the normal to the segment and the reference axis at $i-th$ boundary. The parameters $e$ and $c$ used in Eq. (32) are defined in Figure 2.

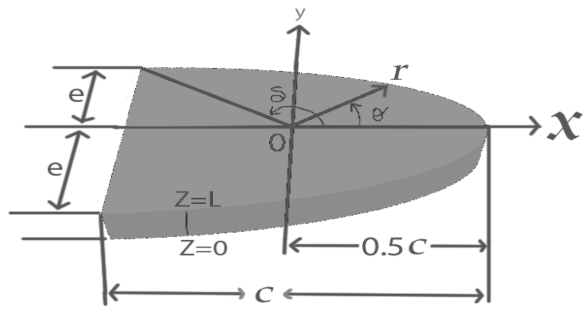


**Figure 2 Parabolic cross-section**

### 6.3 Longitudinal mode

The geometrical relations for the elliptic cross-sections given in Eq. (31) are used directly for the numerical calculations, and three kinds of basic independent modes of wave propagation are studied. In case of the longitudinal mode of elliptical cross-section, the cross-section vibrates along the axis of the plate, so that the vibration and displacements in the cross-section is symmetrical about both major and minor axes. Hence, the frequency equation is obtained by choosing both terms of $n$ and $m$ as $0, 2, 4, 6, \ldots$ in Eq. (26) for the numerical calculations. During the longitudinal motion of parabolic and cardioidal cross-sections, the vibration and displacements are symmetrical about the major axis and hence the frequency equation is obtained from Eq. (26) by taking $n, m = 0,1,2,3\ldots$.

Since the boundary of the cross-sections namely, elliptic, cardioid and parabolic are irregular in shape, it is difficult to satisfy the boundary conditions along the curved surface, and hence Fourier expansion collocation method is applied. In this method, the curved surface, in the range $\theta = 0$ and $\theta = \pi$ is divided into 20 segments, such that the distance between any two segments is negligible and the integrations is performed for each segment numerically by using the Gauss five point formula .The non-dimensional frequencies are computed for $0 < \Omega \leq 1.0$, using the bi-section method (applicable for the complex roots Antia [28])

### 6. 4 Flexural mode

In the case of flexural mode of elliptical cross-section, the vibration and displacements are anti symmetrical about the major axis and symmetrical about the minor axis. Hence, the frequency equations are obtained from Eq. (28) by choosing $n, m = 1,3,5,\ldots$. Since the vibration and displacements are anti symmetrical about the major axis for the parabolic and cardioidal cross-sectional plates, the frequency equation is obtained by taking $n, m = 1,2,3\ldots$ in Eq. (28).

The geometric relations for the elliptic and parabolic cross-sections are given respectively in the Eqs. (31) , (32) and for the cardioids cross-sectional plate, the geometric relations are considered from the Eqs. (24) and (26) of the reference Nagaya [3] are used for the numerical calculations. The notations namely, $S_1, S_2, S_3...$ and $A_1, A_2, A_3...$ used in the graphs and Table respectively represents the symmetric and anti symmetric modes vibration, and the subscripts 1, 2, 3 etc.. represents the first, second, third, fourth and fifth modes vibrations.

The frequency equation for the solid circular plate is obtained by exact method is given in Eq. (30), from the equation., the frequencies are obtained by exact method are used to compare the results of the present method. The dimensionless frequencies are computed for $0 < \Omega \leq 1.0$ for different aspect ratios a/b=1.0, 1.5 and 2.0 for longitudinal and flexural anti symmetric modes of vibrations using bi-section method. The frequencies obtained for the longitudinal and flexural anti symmetric modes of vibration by the exact method is matches well with the frequencies obtained for the aspect ratio a/b=1.0 for a elliptic cross-sectional plate is given in the Table 2.The non-dimensional frequency of elliptic cross section for the aspect ratio a/b=1.0 will represent a circular plate. Since the frequency for the elliptic cross section for the aspect ratio a/b=1.0 matches with the frequency obtained by the exact method is shown in the Table 2. So the problem is extended for elliptic, cardioids and parabolic cross sectional plates.

A graph is drawn between the geometric ratio L/a=0.5 with the aspect ratio a/b=1.5 versus non-dimensional frequency $|\Omega|$ for a longitudinal mode of electro-magneto-elastic plate of elliptic cross-sectional plate is shown in Fig.3. From the Fig.3, it is observed that the non-dimensional frequency increases for different modes of vibrations. The similar behavior is observed for a flexural anti symmetric modes of elliptic cross-sectional electro-magneto-elastic plate is shown in Fig.4. A dispersion curve is drawn between the geometrical ratio L/a=0.5 with the aspect ratio a/b= 0.5, 1.0, 1.5 and 2.0 versus the non-dimensional frequency $|\Omega|$ for a longitudinal modes of electro-magneto-elastic plate of elliptic cross-sectional plate is shown in Fig.5. From the Fig. 5, it is observed that the non-dimensional frequencies are decreases by increasing the aspect ratios, this is the proper physical behavior of plate with respect to its aspect ratios.

The Figs.6 and 7 respectively represents the relation between the geometric ratio L/a=0.5 with geometric parameter s= 0.0, 0.5 and the non-dimensional frequency $|\Omega|$ for a longitudinal and flexural antisymmetric modes of a cardioid cross-sectional plate. From the Figs.6 and 7 it is observed that the non-dimensional frequency $|\Omega|$ increases linearly with respect to the Geometric ratio L/a in both longitudinal and flexural antisymmetric modes of vibrations. The cross-over points shown in the longitudinal modes of cardioidic cross sectional plates represents the transfer of electro-magnetic energy between the modes of vibration.

A graph is drawn between the geometric ratio L/a=0.5 with e/c=1.5 versus non-dimensional frequency $|\Omega|$ for a longitudinal mode of electro-magneto-elastic plate of parabolic cross-sectional plate is shown in Fig.8. From the Fig.8, it is observed that the non-dimensional frequency increases first and then it starts to decreases for a particular period. The cross-over points between the waves represents the transfer of electro-magnetic energy between the modes of vibration. The similar behavior is observed for a flexural antisymmetric modes of parabolic cross-sectional electro-magneto-elastic plate is shown in Fig.9.

## 7. Conclusions

In this paper, the wave propagation in a electro-magneto-elastic plate of arbitrary cross section are analyzed by satisfying the boundary conditions on the irregular boundary using the Fourier expansion collocation method and the frequency equations for the longitudinal and flexural anti symmetric modes of vibrations are obtained. Numerically the frequency equations are analyzed for the plate of different cross-sections such as elliptic, cardioids and parabolic cross sectional plates. The computed dimensionless frequencies are plotted in graphs for longitudinal and flexural anti symmetric modes of vibrations. The problem can be analyzed for any other cross-section by using the proper geometric relation.

## 8. Acknowledgement

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## Appendix A

$$\begin{aligned}
e_n^i = & [-2\bar{c}_{66}\cos 2(\theta-\gamma_i)\{n(n-1)J_n(\alpha_i ax)+(\alpha_i ax)J_{n+1}(\alpha_i ax)\} \\
& + x^2[(\alpha_i a)^2(\bar{c}_{11}\cos^2(\theta-\gamma_i)+\bar{c}_{12}\sin^2(\theta-\gamma_i)) \\
& + t_L(\bar{c}_{13}a_i+\bar{e}_{31}b_i+\bar{q}_{31}c_i)J_n(\alpha_i ax)]\cos(t_L)\cos n\theta \\
& -2n\bar{c}_{66}\{n(n-1)J_n(\alpha_i ax)-(\alpha_i ax)J_{n+1}(\alpha_i ax)\}\sin 2(\theta-\gamma_i)\cos(t_L)\sin n\theta, i=1,2,3,4
\end{aligned} \tag{A1}$$

$$\begin{aligned}
e_n^5 = & [2\bar{c}_{66}\cos 2(\theta-\gamma_i)n\{(n-1)J_n(a_5 ax)-(a_5 ax)J_{n+1}(a_5 ax)\}\cos(t_L)\cos n\theta \\
& +\bar{c}_{66}[2\{n(n-1)J_n(a_5 ax)+(a_5 ax)J_{n+1}(a_5 ax)\}-(a_5 ax)^2 J_n(a_5 ax)]\cos(t_L)\sin n\theta\sin 2(\theta-\gamma_i)
\end{aligned} \tag{A2}$$

$$\begin{aligned}
f_n^i = & \left[-2\{n(n-1)J_n(\alpha_i ax)+(\alpha_i ax)J_{n+1}(\alpha_i ax)\}+(\alpha_i ax)^2 J_n(\alpha_i ax)\right]\cos(t_L)\cos n\theta\sin 2(\theta-\gamma_i) \\
& +2n\{(n-1)J_n(\alpha_i ax)-(\alpha_i ax)J_{n+1}(\alpha_i ax)\}\cos(t_L)\sin n\theta\cos 2(\theta-\gamma_i), i=1,2,3,4.
\end{aligned} \tag{A3}$$

$$f_n^5 = 2n\left\{(n-1)J_n(a_5ax)-(a_5ax)J_{n+1}(a_5ax)\right\}\cos(t_L)\cos n\theta\sin 2(\theta-\gamma_i)$$
$$+\left[-2\left\{n(n-1)J_n(a_5ax)+(a_5ax)J_{n+1}(a_5ax)\right\}+(a_5ax)^2J_n(a_5ax)\right]\cos(t_L)\sin n\theta\cos 2(\theta-\gamma_i) \quad \text{(A4)}$$

$$g_n^i = \left[(t_L+a_i)\cos(\theta-\gamma_i)+\bar{e}_{15}b_i+\bar{q}_{15}c_i\right]\left\{nJ_n(\alpha_iax)-(\alpha_iax)J_{n+1}(\alpha_iax)\right\}\sin(t_L)\cos n\theta$$
$$+(t_L+a_i)nJ_n(\alpha_iax)\sin(t_L)\sin n\theta\sin(\theta-\gamma_i), i=1,2,3,4. \quad \text{(A5)}$$

$$g_n^5 = -nt_LJ_n(a_5ax)\sin(t_L)\cos n\theta\cos(\theta-\gamma_i)$$
$$-\left\{nJ_n(a_5ax)-(a_5ax)J_{n+1}(a_5ax)\right\}t_L\sin(t_L)\sin n\theta\sin(\theta-\gamma_i) \quad \text{(A6)}$$

$$h_n^i = \left[\bar{e}_{15}(t_L+a_i)-\bar{\varepsilon}_{11}b_i-\bar{m}_{11}c_i\right]\left\{nJ_n(\alpha_iax)-(\alpha_iax)J_{n+1}(\alpha_iax)\right\}\sin(t_L)\cos n\theta, i=1,2,3,4 \quad \text{(A7)}$$

$$h_n^5 = -\bar{e}_{15}nt_LJ_n(a_5ax) \quad \text{(A8)}$$

$$i_n^i = \left[\bar{q}_{15}(t_L+a_i)-\bar{m}_{11}b_i-\bar{\mu}_{11}c_i\right]\left\{nJ_n(\alpha_iax)-(\alpha_iax)J_{n+1}(\alpha_iax)\right\}, i=1,2,3,4. \quad \text{(A9)}$$

$$i_n^5 = -\bar{q}_{15}nt_LJ_n(a_5ax) \quad \text{(A10)}$$

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| $c_{11}$ | $c_{12}$ | $c_{13}$ | $c_{33}$ | $c_{44}$ | $c_{66}$ |
|---|---|---|---|---|---|
| 218 | 120 | 120 | 215 | 50 | 49 |

| $e_{15}$ | $e_{31}$ | $e_{33}$ | $q_{15}$ | $q_{31}$ | $q_{33}$ |
|---|---|---|---|---|---|
| 0 | -2.5 | 7.5 | 200 | 265 | 345 |

| $\varepsilon_{11}$ | $\varepsilon_{33}$ | $\mu_{11}$ | $\mu_{33}$ | $m_{11}$ | $m_{33}$ |
|---|---|---|---|---|---|
| 0.4 | 5.8 | -200 | 95 | 0.0074 | 2.82 |

Units: $c_{ij}\left(10^{9}N/m^{2}\right), \varepsilon_{ij}\left(10^{-9}C/Vm\right), e_{ij}\left(C/m^{2}\right)$
$q_{ij}\left(N/Am\right), \mu_{ij}\left(10^{-6}Ns^{2}/C^{2}\right), m_{ij}\left(10^{-9}Ns/VC\right)$

**Table 2.**Comparison between the frequencies |Ω| obtained from the exact and present methods of longitudinal and flexural anti symmetric modes of vibrations

| Aspect ratio | a/b | 1.0 | | 1.5 | 2.0 |
|---|---|---|---|---|---|
| | Mode | Exact Method | Present Method | Present Method | Present Method |
| Longitudinal Mode | S1 | 1.4772 | 1.4772 | 0.1402 | 0.1362 |
| | S2 | 1.5573 | 1.5573 | 0.6871 | 0.6769 |
| | S3 | 1.8067 | 1.8067 | 1.3589 | 1.2987 |
| | S4 | 2.2159 | 2.2159 | 2.2352 | 2.1058 |
| | S5 | 2.7243 | 2.7243 | 2.7729 | 2.6179 |
| Flexural anti symmetric Mode | A1 | 0.2829 | 0.2829 | 0.2812 | 0.1924 |
| | A2 | 0.8491 | 0.8491 | 0.8434 | 0.6971 |
| | A3 | 1.5583 | 1.5583 | 1.5326 | 1.4982 |
| | A4 | 2.4632 | 2.4632 | 2.4471 | 2.4059 |
| | A5 | 2.9709 | 2.9709 | 2.8862 | 2.7166 |

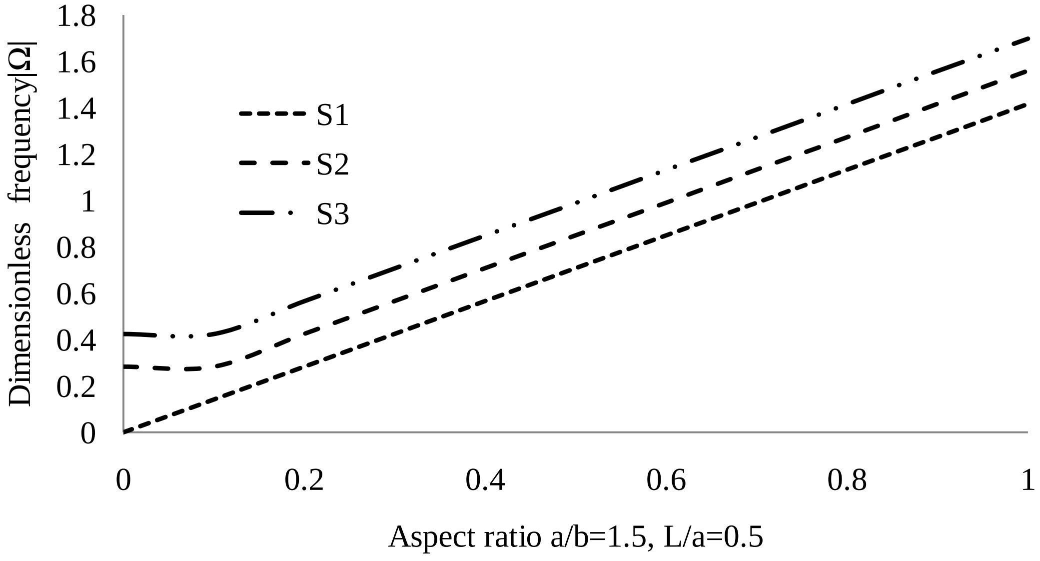
Dimensionless frequency|Ω|
S1
S2
S3
Aspect ratio a/b=1.5, L/a=0.5

Fig. 3

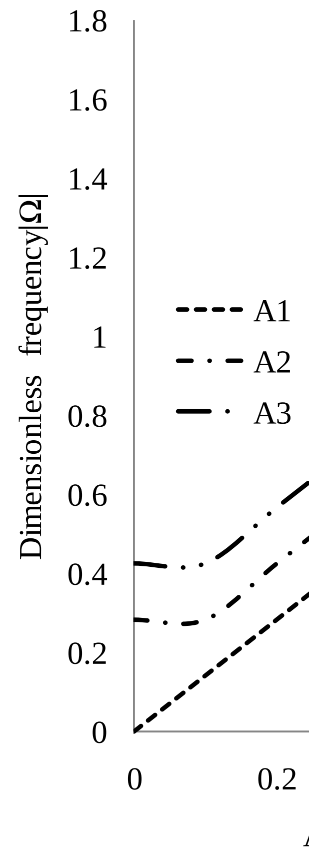
Dimensionless frequency|Ω|
A1
A2
A3

Fig. 4

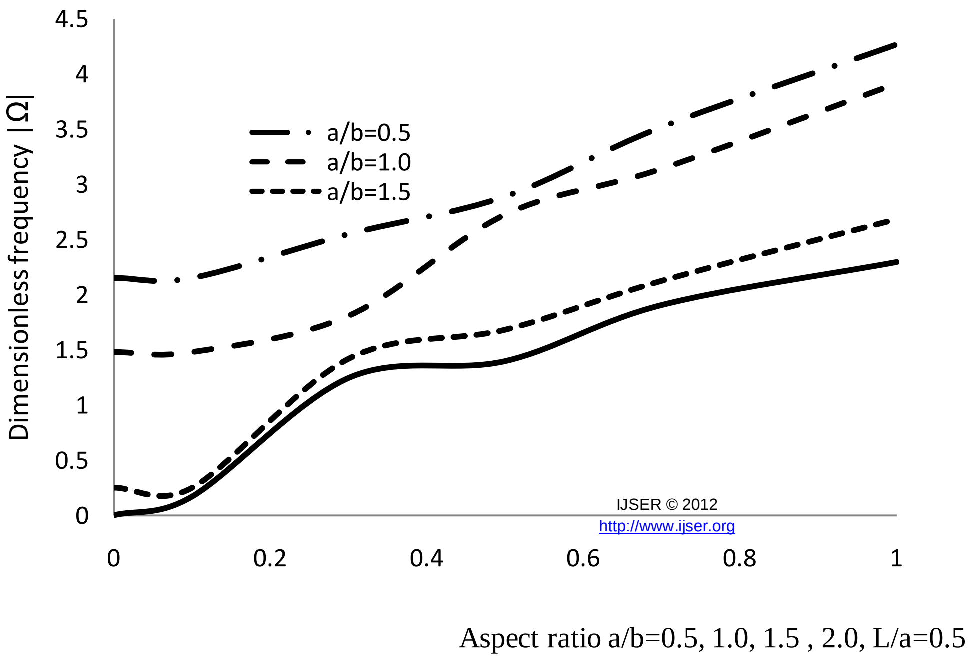
Dimensionless frequency |Ω|
a/b=0.5
a/b=1.0
a/b=1.5
Aspect ratio a/b=0.5, 1.0, 1.5 , 2.0, L/a=0.5

Fig.5

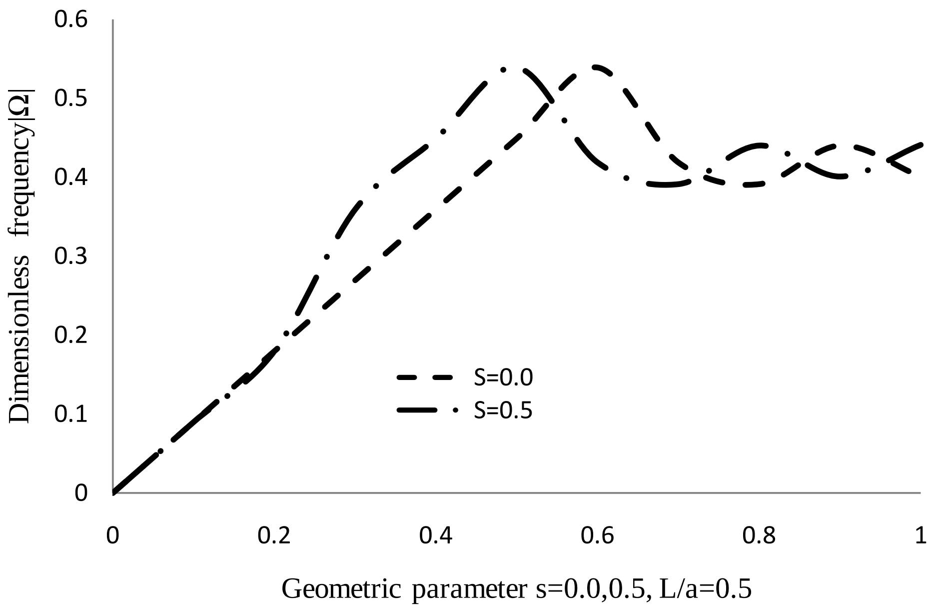


Fig.6

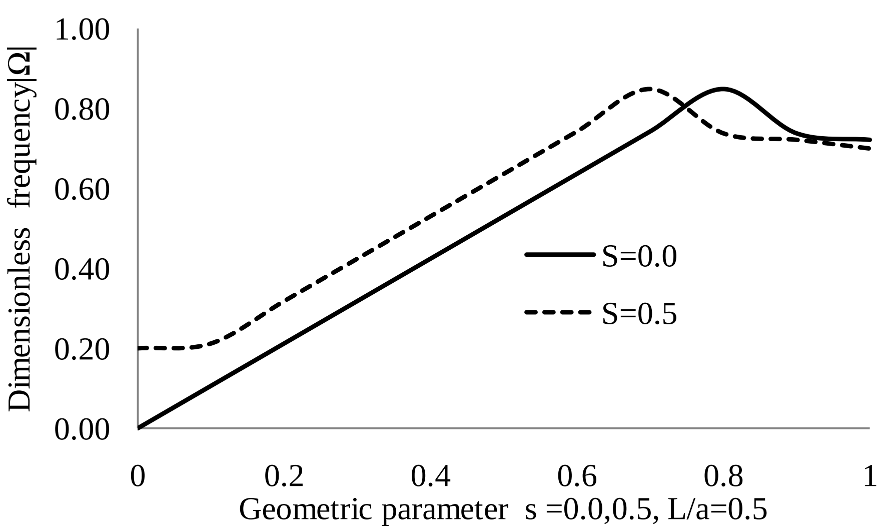


Fig.7

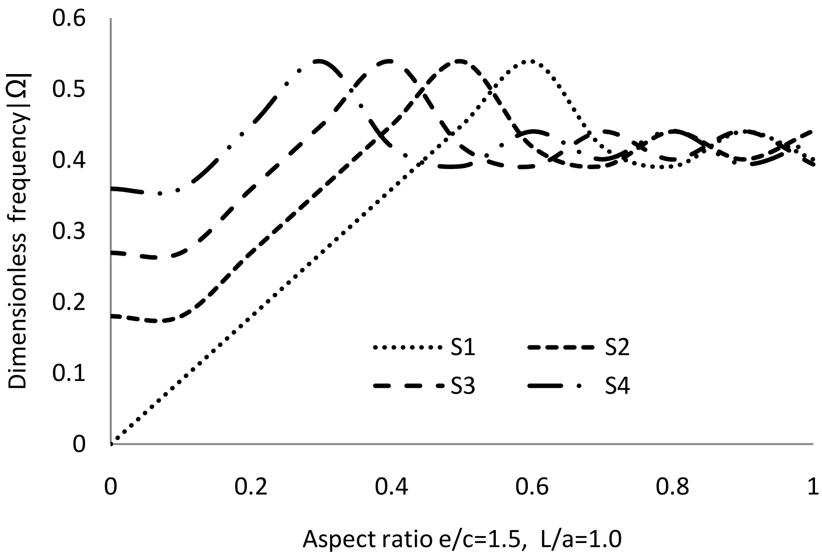


Fig.8

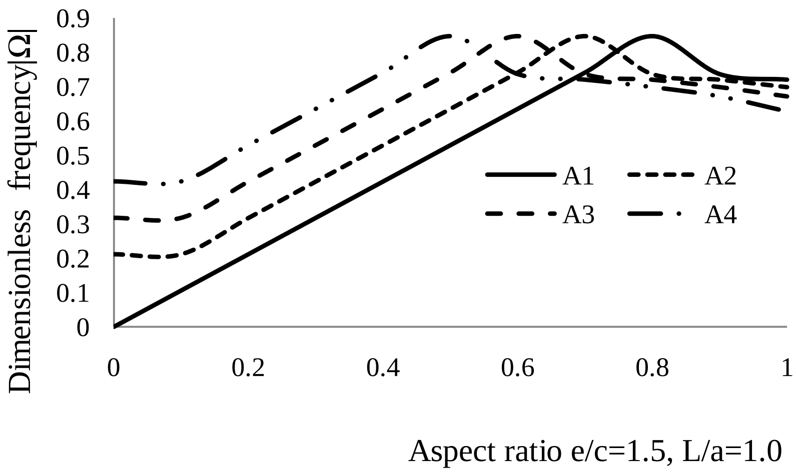


Fig.9

Table Caption

**Table. 1** The material properties of the electro-magnetic material based on graphical results of Aboudi[27] composites

**Table 2.**Comparison between the frequencies |Ω| obtained from the exact and present methods of longitudinal and flexural anti symmetric modes of vibrations

Figure Captions

Fig. 3 Aspect ratio a/b=1.5, L/a=0.5 versus dimensionless frequency|Ω| for longitudinal modes of elliptic cross-sectional plate

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Fig. 4 Aspect ratio a/b=1.5, L/a=0.5 versus dimensionless frequency|Ω| of flexural antisymmetric modes of elliptic cross-sectional plate

Fig. 5 Aspect ratio a/b=0.5, 1.0, 1.5, 2.0, L/a=0.5 versus dimensionless frequency|Ω| for longitudinal modes of elliptic cross-sectional plate

Fig.6 Geometric parameter s =0.0,0.5, L/a=0.5 versus dimensionless frequency|Ω| for longitudinal modes of cardioidal cross-sectional plate

Fig.7 Geometric parameter s =0.0,0.5, L/a=0.5 versus dimensionless frequency|Ω| for flexural anti symmetric modes of cardioidal cross-sectional plate

Fig. 8 Aspect ratio e/c=1.5 with L/a=1.0 versus dimensionless frequency|Ω| of longitudinal modes of parabolic cross-sectional plate

Fig. 9 Aspect ratio e/c=1.5 with L/a=1.0 versus dimensionless frequency|Ω| of flexural antisymmetric modes of parabolic cross-sectional plate